Paper Type: AD

M. Prakash Academy Entrance Examination 2015

Standard IX

19^{th} April 2015. Time: 10.00 am to 12.30 pm Total marks: 200

Student's Name: Receipt Number:

Mathematics

Q1. Virat runs twice as fast as he walks. He travels from his house to school by walking some distance and by running some distance. On Monday his walking time is twice his running time and reaches the school in 30 minutes. On Tuesday his running time is twice his walking time. Find the time in minutes he takes to reach the school on Tuesday.

Q2. The fraction $\frac{35}{16}$ can be written in the form $\frac{35}{16} = 2 + \cfrac{1}{x + \cfrac{1}{y}}$ where x, y are natural numbers. Find $(x+y)^2$.

Q3. Let $n = 100^{25} - 25$. Let S denote the sum of the digits of n. Find the smallest natural number k such that $S + k$ is a perfect square.

Q4. If $1.236 \times 10^{15} - 5.23 \times 10^{14} = a.bc \times 10^k$ where a, b, c are digits from 1 to 9 and k is a natural number. Find $(a + b + c + k)$.

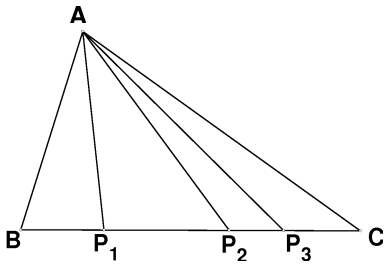
Q5. Two adults have their birthday on the same day. On a particular birthday the product of their ages is 770. Find the sum of their ages on that birthday.

Q6. Mohit bought a number of balls. He was required to pay 5% tax on his purchase. If he did not have to pay the tax he could have bought 3 more balls in the total amount he had spent. How many balls did Mohit buy?

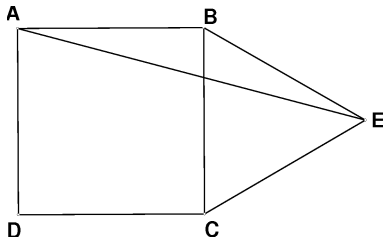
Q7. Two candles of length one feet each start burning at the same time. One of the candles will burn down in 40 hours and the other in 24 hours. If after H hours one of the candle has length 3 times the length of the other candle, find H.

Q8. 125 small cubes of size $1 \times 1 \times 1$ are put together to form a cube of size $5 \times 5 \times 5$. Two cubes of size $1 \times 1 \times 1$ are said to be neighbours if they are placed such that their one face of size 1×1 touches each other. Find the number of $1 \times 1 \times 1$ cubes having exactly four neighbours.
(Note that a cube has 8 vertices, 12 edges and 6 faces.)

Q9. In $\triangle ABC$, three points P_1, P_2, P_3 are placed on segment BC and each joined to vertex A. The resulting figure contains all together 10 triangles. Find the total number of triangles present in the figure if 12 points $P_1, P_2, \cdots, P_{11}, P_{12}$ are placed on segment BC and each is joined to vertex A.

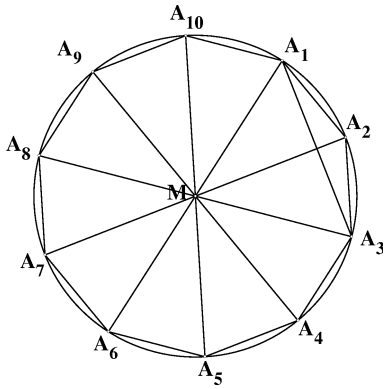


Q10. As shown in the figure $\square ABCD$ is a square and $\triangle EBC$ is an equilateral triangle. Find the measure of $\angle DAE$.



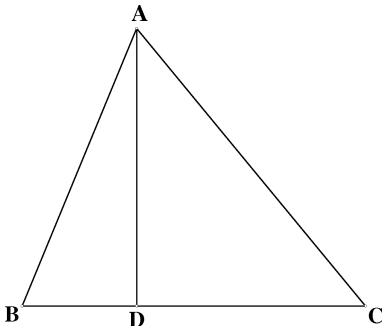
Q11. A polygon $A_1A_2A_3.....A_{10}$ of 10 sides is inscribed in a circle, that is all the vertices lie on the circumference of the same circle. All the 10 sides are of equal length. Let M be the center of the circle.

Find the measure of $\angle A_3A_1A_2$.

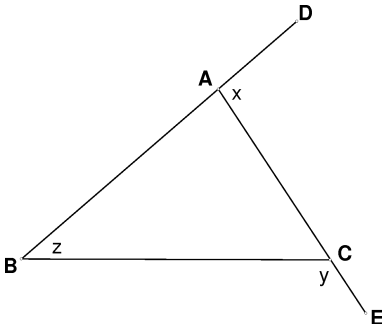


Q12. In $\triangle ABC$ D is on segment BC such that $BD = x$ and $DC = 2x$. M is the midpoint of segment AC. If area of $\triangle ABD$ is 26 units find the area of $\triangle BMA$.

Q13. In $\triangle ABC$, D is the foot of the altitude from A on segment BC. If $AD = 24$, $AC = 30$ and $BC = 28$. Find AB.



Q14. As shown in the figure $m\angle DAC = x°, m\angle BCE = y°$ and $m\angle ABC = z°$. Find $\frac{1}{12}(x + y - z)$.



Q15. I have a 14 digit interesting number. The sum of its any three consective digits is same. If its first digit is 4, its 5^{th} digit is 7 and the sum of its all digits is 79 then find the sum of its last 4 digits.

Q16. Let $n = (7584)^2 + 4(7584)(1208) + 4(1208)^2$. Find the smallest value of the natural number m such that the product of m and n is a perfect cube.

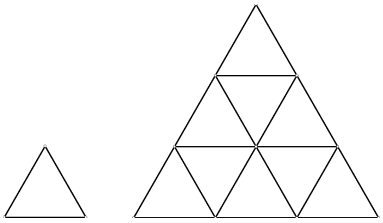
Q17. Let m be the largest divisor of 72^3 other than itself. Let n be the largest divisor of 75^4 other than itself. If L.C.M. of m and $n = p^a q^b r^c$ where p, q, r are distinct prime numbers then find the value of $a + b + c$.
(Note that the largest divisor of 10^3 other than 10^3 is 500).

Q18. We define a new operation $*$ as given below.
$a * b = a^2 + b$. For example, $8 * 5 = 8^2 + 5 = 69$.
If n is a natural number and p is a prime number then find the smallest value of p satisfying $8 * p = n * 6$.

Q19. Find the value of $(2\sqrt{20} + 4\sqrt{8})(6\sqrt{2} - \sqrt{45})$.

Q20. If $a^2 - b^2 = 132$ and $a + b = 22$ then find the value of $a + b^2$.

Q21. As shown in the following figure equilateral triangle of size 1 is formed using 3 match sticks. To form an equilateral triangle of size 3 with all the equilateral triangle of size 1 inside it we require 18 match sticks. Find the total number of match sticks required to construct a size 6 equilateral triangle with all the equilteral triangle of size 1 inside it.



Q22. Consider the 8-digit number $3681m42n$ where m and n are digits from 0 to 9. Find the value of $m^2 + n$ if given 8-digit number is divisible by 72.

Q23. Find the value of $100(1 - \frac{1}{8})(1 - \frac{1}{9})(1 - \frac{1}{10})\ldots\ldots\ldots(1 - \frac{1}{20})$.

Q24. Integer k is 4^{th} power of another integer. If 18 is a factor of k then find the smallest value of $\frac{k}{18}$.

Q25. Club G has several members. Average age of members of G increases by one year if either five members each 9 year old leave G or new five members each 17 year old join G. Find the present number of members in G.

Science

Some useful information:

I - Whenever you are asked to write only integer part of an answer, you need to enter the portion on the left hand side of the decimal point. For example, if your answer is 28.34, you should enter 28 in your bubble sheet. If your answer is 28.94, even in this case enter 28 in your bubble sheet.

II - Some useful constants are:

Atomic numbers - Hydrogen (H) = 1, Carbon (C) = 6, Nitrogen (N) = 7, Oxygen (O) = 8

Atomic weights - H: 1, C = 12, N= 14, O = 16

Q26. A merchant sells mangoes and apples in his shop. Selling price of one mango is $40Rs.$ and that of apple is $20Rs.$ On one day, he has 100 fruits when he opens the shop. At end of the day he is left with 65 fruits and has collected $Rs.900$. **Find out number of apples sold and enter that number in your bubble sheet.**

Q27. . The Sun is extremely massive compared to the earth. It is 330000 times heavier than the earth i.e. 1 solar mass = 330000 earth mass. Mass of earth is $5.98 \times 10^{24}\ Kg$. Calculate the mass of the Sun. Express your answer as $A \times 10^{B}\ Kg$, where A is less than 10 and B **is a natural number. Enter the value of** B **in your bubble sheet.**

Q28. Following are the details of motion of an object over period of 1 minute. It is initially at point A. After 10 seconds, it starts moving with constant acceleration to the right and reaches point B in 20 seconds. It travels with constant speed for further 10 seconds to reach point D. It then decelerates until completion of 1 minute to stop at point E. Points A, B, C, D and E lie on a straight line.
During this motion, there is **some duration of time for which there is no net force acting on the object. Find this duration in seconds and enter that number in your bubble sheet.**

Q29. Consider an object of mass 9 Kg in motion. Its initial velocity is 24 m/s and it slows down to a final velocity of 9 m/s in 3 seconds. Calculate the **net force acting on the object. Express your answer in Newton.**

Q30. Pressure is measured at three locations. Pressure A is measured at a point one meter below the surface of a small pond. Pressure B is measured at a point one meter below the surface of a huge lake. Pressure C is measured at a point one meter below the surface of a water tank. **Which of following statements is true? Enter the number of that statement in your bubble sheet.**

(11) $A > B > C$

(33) $C < A < B$

(55) $B < A < C$

(77) $C = A = B$

Q31. A neutral plastic strip is rubbed with cotton. The strip acquires a positive charge. Following are some of the statement about the positively charged strip. **Some of the statements are true and some are false. Read these statements carefully and choose the correct option. Enter the number of that option in your bubble sheet.**

A. The strip lost some electrons to the cotton during the charging process.

B. The strip lost all of its electrons to the cotton during the charging process.

C. The strip has the opposite charge as that on the cotton.

D. The strip would now be repelled by the piece of cotton which was used to charge it.

E. The strip gained protons during the rubbing process.

F. As a material, plastic has a greater affinity for electrons than cotton.

G. The strip could exert either a repulsive or attractive influence upon neutral paper bits.

H. The strip has an excess of protons compared to the number of electrons.

I. The strip lost negative electrons and gained positive electrons during the charging process.

J. The strip lost neutrons during the charging process (or at the very least, its neutrons became ineffective).

Options

(11) Statements B, C, E and H are true.

(33) All statements are true and only statement I is false.

(55) Statements A, C and H are true and rest are false.

(77) A, C, E, G, H and J are true.

Q32. A bulb of 3 *cm* diameter is 50 *cm* away from a screen. A ball with a diameter of 3 *cm* is held exactly halfway between the bulb and the screen. An umbra and a penumbra are formed on the screen. What will be the **diameter of the umbra region of the shadow? Express your answer in centimeter.**

Q33. A flask contains liquefied mixture of four substances A, B, C and D having boiling points as shown in the table. This mixture is subjected to fractional distillation so that the constituents are separated one by one. **Which substance will be left behind in the flask after completion of the process? Write the number assigned to that substance (number from the third column.)**

Substance	Boiling Point (^0C)	Number assigned
A	-182	32
B	-35	34
C	-57	44
D	-196	28

Q34. To find the percentage of an element in a particular compound, the following steps are followed:

$$\text{The \% of an element in a compound} = \frac{\text{weight of the element}}{\text{molecular weight of compound}} \times 100$$

[For example : Molecular weight of H_2O : $2\times$ atomic weight of Hydrogen $(1) + 1\times$ atomic weight of oxygen $(16) = 18$.
Percentage of Hydrogen in water $= \frac{2}{18} \times 100 = 11.11\%$]

Now attempt the following problem.

Ammonium nitrate - NH_4NO_3 and Urea - $CO(NH_2)_2$ are two examples of fertilizers containing Nitrogen. **Find the percentage of Nitrogen in that fertiliser which is richer in Nitrogen percentage. Write the integer part of your answer in the bubble sheet.**

Q35. Balance the following chemical equation **by replacing a, b, c, d and e with appropriate natural numbers.**

$aS + bHNO_3 \rightarrow cH_2SO_4 + dNO_2 \uparrow + eH_2O$

Write the sum of a, b, c, d and e in your bubble sheet.

Q36. Most abundant element in air reacts with lightest element present in nature at high pressure and low temperature, in the presence of iron as a catalyst. A pungent smelling gas is produced. **Find the molecular mass of this gas.**

Q37. Consider 60 gm sample of 12 Carat Gold. There is another sample, 90 gm of 18 Carat gold. These two samples are mixed together to create an impure alloy. **Find the percent purity of the new alloy.**
Given: 24 Carat = 100% pure gold.

Q38. Carbon has two isotopes, Carbon-12 and Carbon-14. **Find the sum of electrons, protons and neutrons present in ^{12}C and ^{14}C isotopes of carbon.**

Q39. Following is a list of some substances:

Cotton, rayon, jute, nylon, wool, terylene, melamine, acrylic, bakelite, polythene, silk, PVC (polyvinyl chloride), polyester.

Find the total number of **natural fibers** in the list. Call it X.
Find the total number of **artificial fibers** in the list. Call it Y.
Find the total number of **plastics** in the list. Call it Z.

Find the value of $(X + Y - Z)$ and enter it in your bubble sheet.

Q40. Following is a list of some substances:

Common salt, naphthalene, limestone, potash alum, iodine, white phosphorous, camphor, sugar and ammonium chloride.

How many of these substances can be separated by the process of sublimation? Enter this number in your bubble sheet.